**MATHEMATICS SPECIALIST**

**MAWA Year 12 Examination 2018**

**Calculator-assumed**

# Marking Key

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The release date for this exam and marking scheme is

* **the end of week 8 of term 2, 2018**

**Question 8 (5 marks)**

|  |  |
| --- | --- |
| Solution | |
| If  then if  we have  .  Since we have assumed that we reject the negative square root.  If then    Since we have assumed that we reject the positive root.  Hence the solution is  or . | |
| Mathematical behaviours | Marks |
| * calculates the correct root assuming that (2 marks) * calculates the correct root assuming that  (2 marks) * states the overall solution correctly | 1  1  1 |

**Question 9 (a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| From the graph it appears that  has a zero at  We evaluate  So by the factor theorem  is a factor of | |
| Mathematical behaviours | Marks |
| * obtains  as a possible zero of *p* * demonstrates that * deduces that  is a factor of | 1  1  1 |

**Question 9(b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Long division gives that  If  by either using the quadratic formula or completing the square  Hence  and  are the conjugate linear factors of | |
| Mathematical behaviours | Marks |
| * identifies the correct quadratic factor * determines correctly the zeros of the quadratic factor * states correctly the corresponding conjugate linear factors | 1  1  1 |

**Question 10(a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * correctly states the horizontal and vertical components of * simplifies correctly | 1  1 |

**Question 10(b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| The acceleration is | |
| Mathematical behaviours | Marks |
| * integrates  to determine * determines the constant and states the correct | 1  1 |

**Question 10(c) (6 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * integrates  to determine  (accept assumption of constants of integration as 0 from initial conditions, without statement) * uses vertical component of  determine the time when max height is reached * determines the max height * recognises that the max horizontal distance when vertical component =0 * determines the time when max distance is reached * determines max distance | 1  1  1  1  1  1 |

**Question 11 (a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Now  and  Since a pair of adjacent sides in the parallelogram *P* have the same length, all sides have the same length.  Hence Pis a rhombus. | |
| Mathematical behaviours | Marks |
| * calculates the correct values of  and * makes a valid deduction about the lengths of all sides of P | 1  1 |

**Question 11 (b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Denote  . Then  Denote  . Then  Hence  So the required angle is 2.21 radians (correct to 2 decimal places) | |
| Mathematical behaviours | Marks |
| * determines correctly the value of * determines correctly the value * gives the required answer to the prescribed accuracy | 1  1  1 |

**Question 11 (c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| We see that  and that  Hence it follows that | |
| Mathematical behaviours | Marks |
| * evaluates correctly the value of * evaluates correctly the value of . | 1  1 |

**Question 11 (d) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| The diagonals of P are  and  .  From part (c) we deduce that …….(i)  Hence one diagonal is twice as long as the other.  Also we have that  ………(ii)  Hence the angle between the diagonals is  . | |
| Mathematical behaviours | Marks |
| * derives the result (i) * deduces the correct ratio of the lengths of the diagonals * derives the result (ii) * deduces the correct angle between the diagonals | 1  1  1  1 |

**Question 12(a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Since the function involves a square root we need to check that the quadratic is positive for all real values of.  As    the function is defined for all real values | |
| Mathematical behaviours | Marks |
| * notes that we need to show that the contents of the square root is positive * either completes the square or shows the quadratic never equals zero * deduces the required result | 1  1  1 |

**Question 12(b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| If  then    This then implies that  or | |
| Mathematical behaviours | Marks |
| * forms an equation without the square root expressing the fact that * solves the equation * states the two possible solutions | 1  1  1 |

**Question 12(c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| The function is not one-to-one.  If the function were 1-1 then would force .  Since here we have the possibility that  we do not have a 1-1 function | |
| Mathematical behaviours | Marks |
| * states the correct conclusion * indicates the property that must be satisfied by a 1-1 function * justifies why this function does not possess this property | 1  1  1 |

**Question 13 (6 marks)**

|  |  |
| --- | --- |
| Solution | |
| If  then  .  Also  so that  for  .  Hence  or  .  Written in polar form,  cis with  or  .  Moreover, cis cis cis cis  and  so the solutions are  . ,  and . | |
| Mathematical behaviours | Marks |
| * derives the result that * deduces the correct value of * obtains one value of * determines all four possible values of * writes down the the real and imaginary parts of the four ‘‘cis’ values * solves correctly for the real and imaginary parts of all four solutions | 1  1  1  1  1  1 |

**Question 14(a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| If  Therefore the inverse function is  (Alternatively students may define the other branch with  so  .) | |
| Mathematical behaviours | Marks |
| * writes the equation * solves for * states correct inverse function | 1  1  1 |

**Question 14(b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Domain is  Range is  (If student took the alternative definition the range becomes  .) | |
| Mathematical behaviours | Marks |
| * states correct domain * states correct range | 1  1 |

**Question 14(c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * gives neat sketch of * gives neat sketch of | 1  1 |

**Question 14(d) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| The two graphs are reflections of each other in the line | |
| Mathematical behaviours | Marks |
| States the correct geometrical relationship with marks   * for mentioning reflection * for giving the equation of the line of reflection | 1  1 |

**Question 15(a)(b) (3+2 marks)**

|  |  |
| --- | --- |
| Solution | |
| since  Point  moves in a horizontal circle, +2 units above the as indicated by the 2.  The centre of the circle is (0,0,2) and the radius is 1 unit.  At  so the particle starts at (1,0,2) and at  so the particle moves in an anticlockwise direction. | |
| Mathematical behaviours | Marks |
| * indicates that the particle moves in plane parallel to the and 2 units above it * indicates that it is a circle * states the centre and radius of the circle * indicates that the particle moves in an anticlockwise direction * provides an appropriately labelled diagram of the circle in roughly the right position | 1  1  1  1  1 |

**Question 15(c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| The particle remains at a constant distance from the origin. | |
| Mathematical behaviours | Marks |
| * determines the correct distance * states that the distance from the origin is constant over time | 1  1 |

**Question 15(d) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| The position vector is given by  Differentiating | |
| Mathematical behaviours | Marks |
| * determines the correct velocity * (allow one mark if attempts to differentiate but makes an error) | 2 |

**Question 15(e) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Given  moves in a circle in the  , with centre (0,2) and radius 2, the vector equation is  with | |
| Mathematical behaviours | Marks |
| * states the correct cartesian equation * remembers to state that | 1  1 |

**Question 15(f) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * indicates the need to determine * determines * calculates the correct distance | 1  1  1 |

**Question 16(a) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| By De Moivre’s theorem we have  .  Expanding and taking real parts gives | |
| Mathematical behaviours | Marks |
| * uses De Moivre’s theorem appropriately * expands the fourth power of the expression correctly * takes the real parts of each side * replaces  and simplifies to obtain the result | 1  1  1  1 |

**Question 16 (b) (5 marks)**

|  |  |
| --- | --- |
| Solution | |
| Let  where  . Then by part (a) we have  Hence the maximum and minimum values of  are  .  At maximum values  so that  or  within the range.  Since we have .  At minimum values  so that  or  within range whence  .  In summary, the maximum value of is 1, and occurs at  and the minimum value of is -1, and occurs at | |
| Mathematical behaviours | Marks |
| * derives 1 and -1 as the extreme values of * determines the correct values of θ at the maximum * infers the corresponding correct values of * determines the correct values of θ at the minimum * infers the corresponding correct values of | 1  1  1  1  1 |

**Question 17(a) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| We have  . Domain is  and range is  (all real numbers)  Similarly . Domain is  and range is (all real numbers) | |
| Mathematical behaviours | Marks |
| * determines the two composite functions correctly (one mark for each) * states domain and range of * states domain and range of | 1  1  1 |

**Question 17(b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| If  then | |
| Mathematical behaviours | Marks |
| * forms an appropriate equation for * solves correctly | 1  1 |

**Question 18(a) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Since plane  is parallel to the , and  everywhere on the plane the equation is  (in vector form the equation is **r.i****)** | |
| Mathematical behaviours | Marks |
| * states the correct equation of the plane | 1 |

**Question 18(b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Largest sphere will have a diameter = 3 units, so the radius = 1.5 units and centre =  Vector form:  Cartesian form: | |
| Mathematical behaviours | Marks |
| * states the centre and radius of the sphere * states the vector equation * states the Cartesian equation | 1  1  1 |

**Question 18(c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * determines the cross product of two appropriate vectors * uses the cross product to determine the equation of the plane * states the correct plane equation | 1  1  1 |

**Question 18(d) (5 marks)**

|  |  |
| --- | --- |
| Solution | |
| 2. Substituting into     so its position vector is **i**+**j** + **k** | |
| Mathematical behaviours | Marks |
| * determines the position vector for * uses the position vectors of  and to determine the vector equation of the line * states the equation of the line in equivalent parametric form. * substitutes into the equation of the plane   (or whatever found in part (c))   * states the position vector of the point of intersection | 1  1  1  1  1 |

**Question 18(e) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Using the dot product of and  or the angle between vectors on a CAS calculator the angle between the line and the plane is 91.6°. | |
| Mathematical behaviours | Marks |
| * determines an appropriate vector in the line (e.g ) * determines an appropriate vector in the plane (e.g) * states the angle between the plane and the line (accept any suitable rounding) | 1  1  1 |